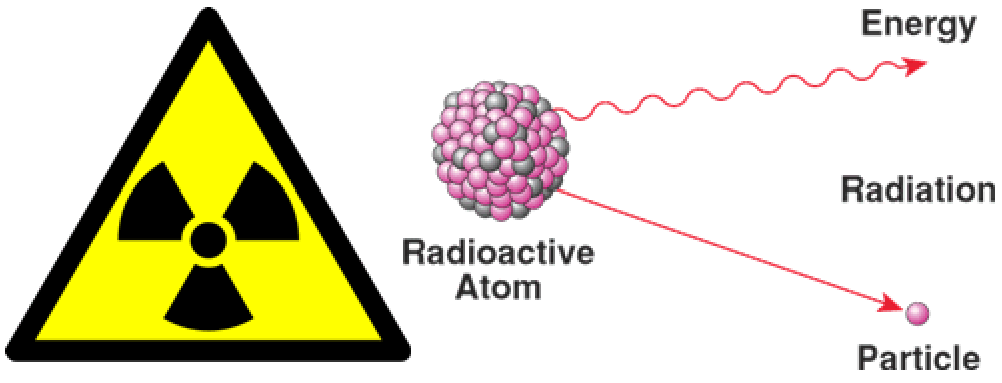
Localization of Radioactive Sources



Zhifei Zhang

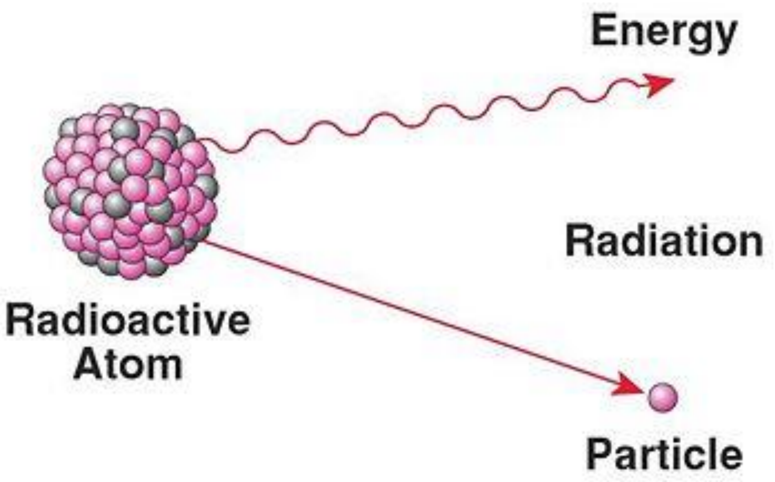
Outline

- **Background and motivation**
- **Our goal and scenario**
- **Preliminary knowledge**
- **Related work**
- **Our approach and results**

Background and Motivation

- For security purpose or searching missing radiological materials, localization of radioactive source is required.
- Many algorithms exist to perform source detection or identification. However, efforts at source localization are limited (e.g., maximum count rate, MLE).
- The detecting output may vary with angle, distance, duration time, and environment (e.g., background, shadow of obstacles).

Background and Motivation

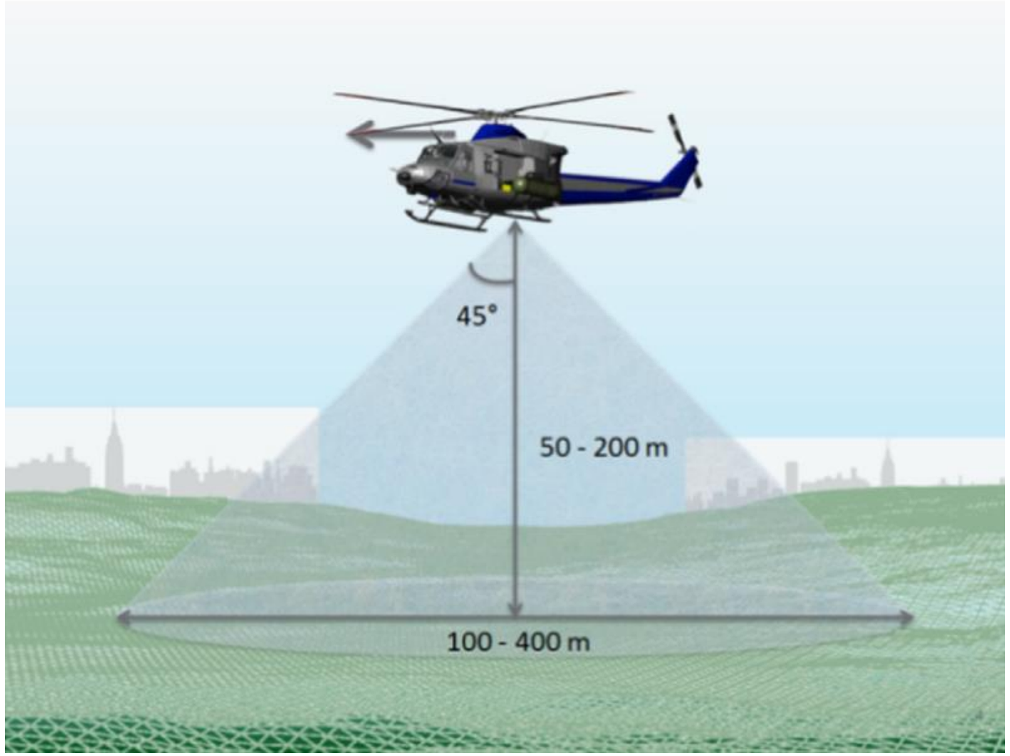


Detector

The detector can be carried by a helicopter, truck, or human.
An naïve way of radioactive source localization is base on **maximum count rate**.

Longer detecting time
→ more particles are captured
→ higher SNR
→ get count rate with higher confidence

Background and Motivation



Aerial detection

Maximum count rate:

Search every corner of the target area to find the location with the maximum count rate.

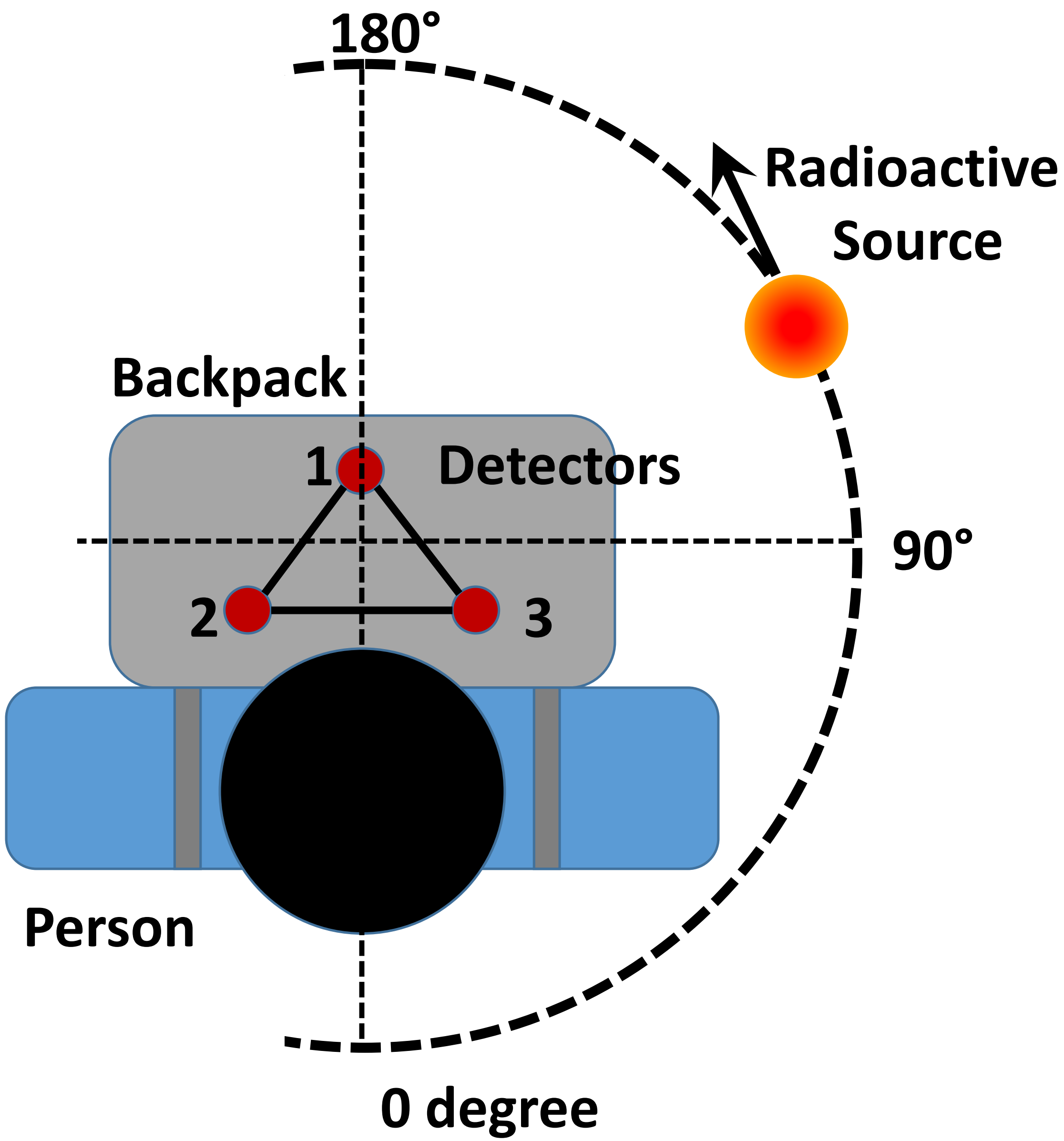
A more efficient way:

Train a model in prior, and then estimate the location by **Maximum Likelihood Estimation** (**MLE**).

Our Goal and Scenario

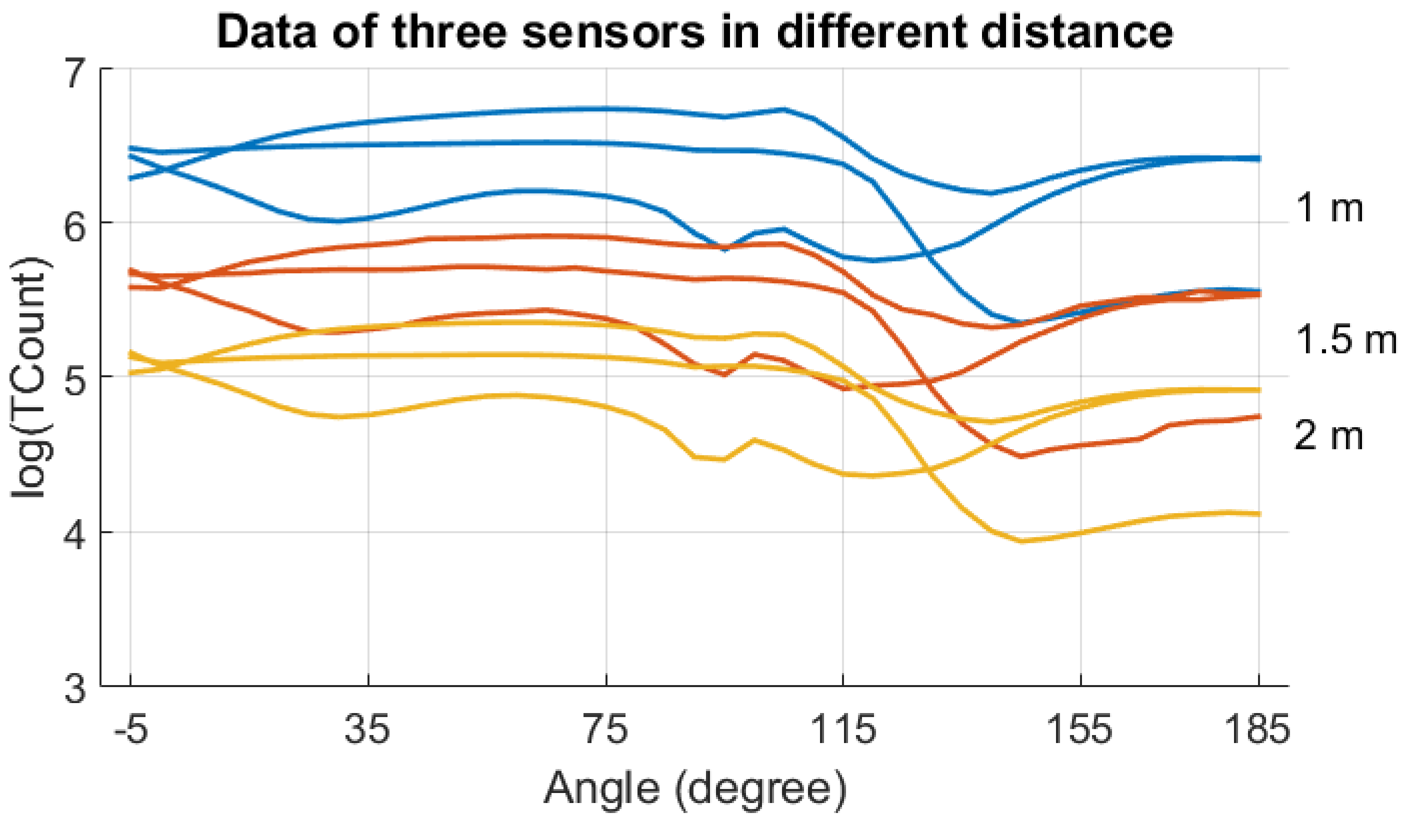
Goal: Localize (angle θ and distance r) the radioactive source through human-carried detector.

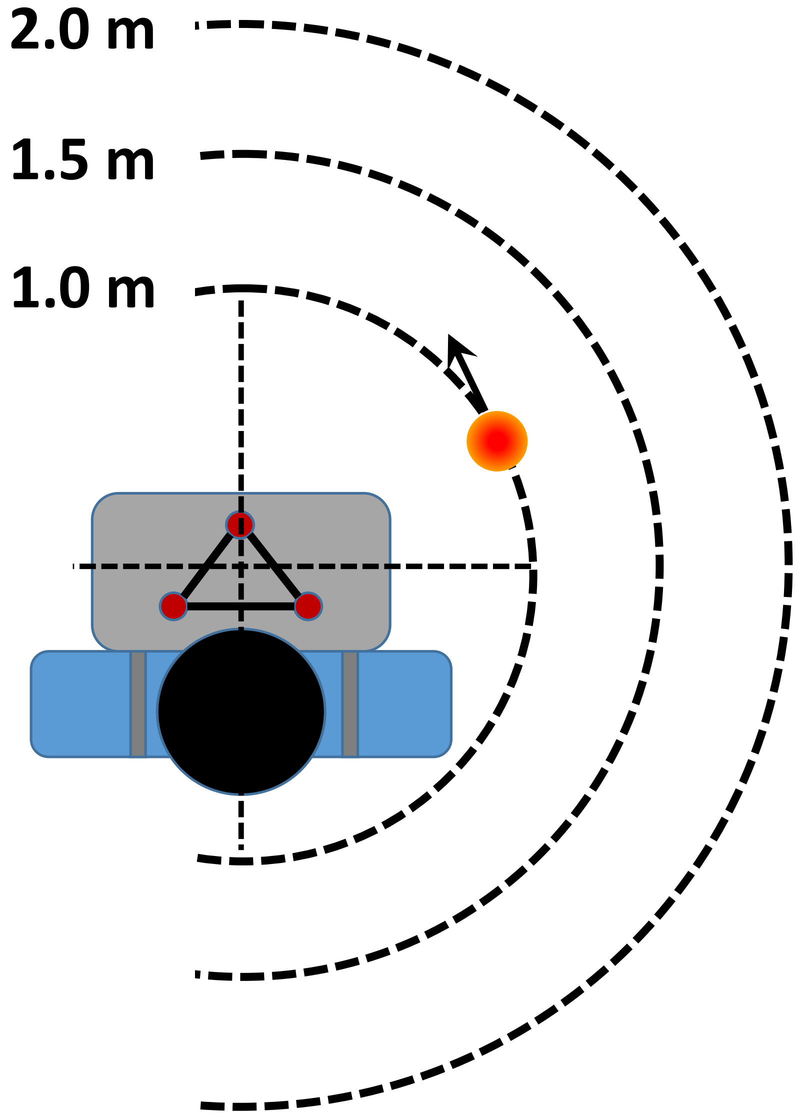
Scenario: A person with a backpack, carrying a group of sensors with certain structure. Assume a radioactive source rotates around the person.



Our Goal and Scenario

Simulation on different distances.



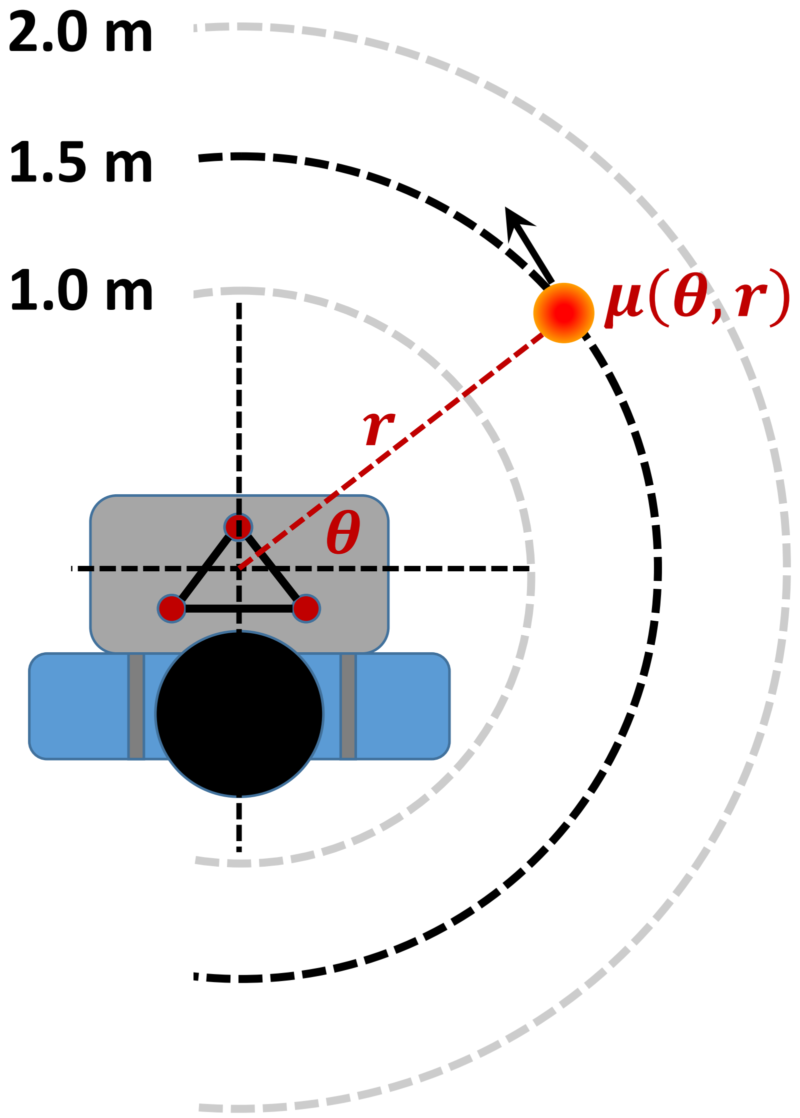


Our Goal and Scenario

Final Goal:

Estimate a model or function of angle θ and distance r, $\mu(\theta, r)$, for each detector, so that count rate of the ith detector equals to $\mu_i(\theta, r)$. Assume an observation of the ith detector at θ and r is T_i, thus

$$T_i \approx \mu_i(\theta, r)$$

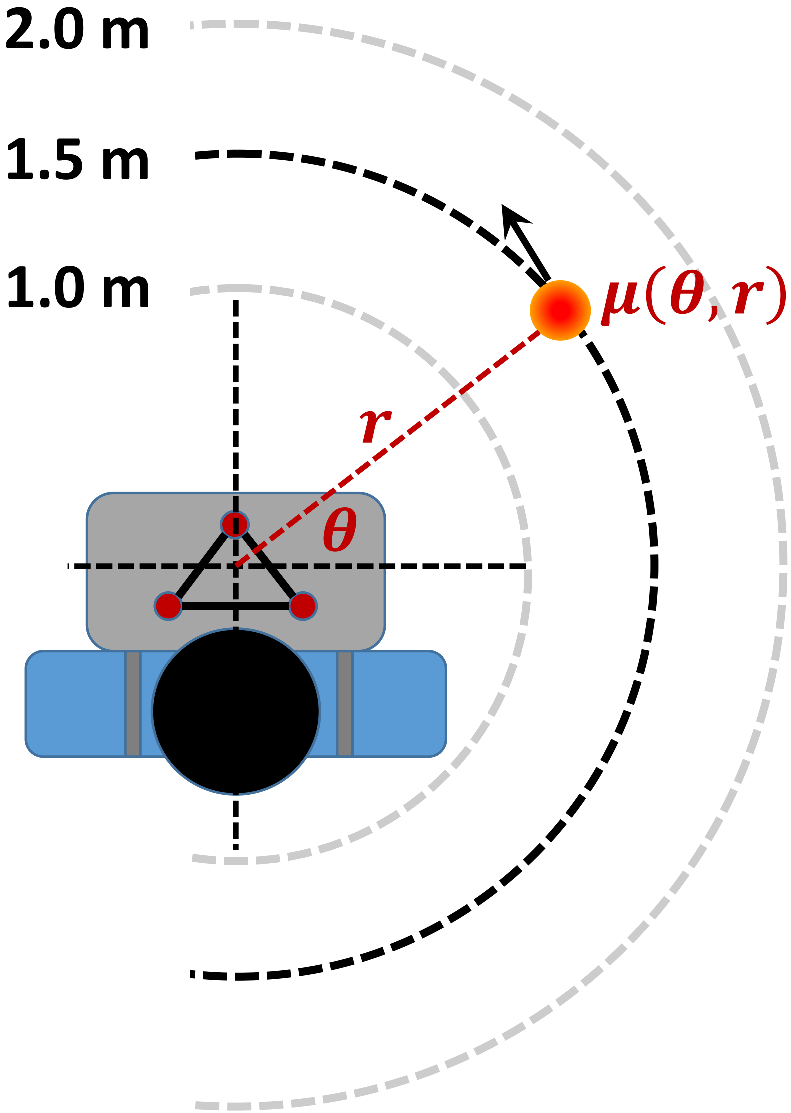


Our Goal and Scenario

In practice, the radioactive source is fixed and the person is moving. Given $\mu(\theta, r)$ and an observation T, the correspond θ and r can be estimated by:

- MLE: $\arg\max_{\theta,r} P\big(T|\mu(\theta,r)\big)$
- 1NN: $\arg\min_{\theta,r} \|T - \mu(\theta,r)\|_2$

(More details later …)



Preliminary Knowledge

Activity: The total number of emission per second in all directions from the source. It is a constant

$$1\text{Ci} \; = \; 3.7 \; \times \; 10^{10}$$

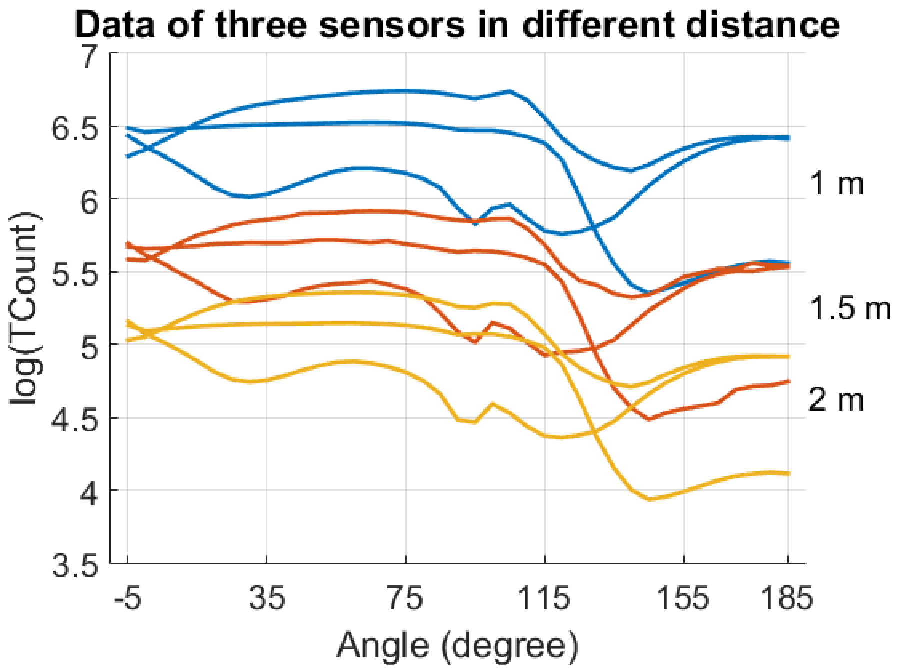
Count rate (T): The number of emissions record by the detector. The observed count rate is always much less than the activity.

$$T = \underbrace{\underset{\text{count per particle}}{\text{count}}}_{\text{Data from simulation}} \times \underbrace{\underbrace{\left(100\times10^{-6}\right)}_{\text{100 times of }\mu\text{Ci}}}_{\text{Recorded particles (constant)}} \times \underbrace{\underbrace{\left(3.7\times10^{10}\right)}_{\text{activity}} \times \underset{\text{detection time in sec}}{1.000}}_{\text{All emitted particles per sec (constant)}}$$

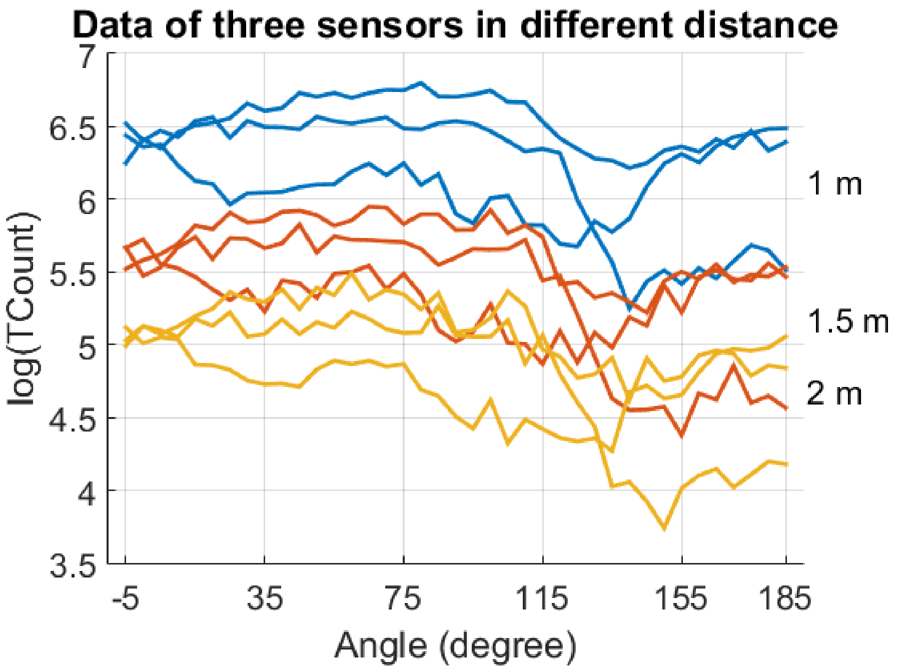
Preliminary Knowledge

Uncertainty: Smaller count rate will result in higher uncertainty.

$$T \sim N(T, \sqrt{T}^{2})$$



Raw signal



Noisy signal

Related Work

Model-free (sensor network):

- Angle-base (Mean of Estimates) [D. Niculescu et al., 2003]
- Distance-based (Apollonius circle) [J.C. Chin et al., 2008]
- Maximum count rate (stationary source) [D.K. Fagan et al., 2012]

Model-based:

Maximum Likelihood Esitmation (MLE) [A. Gunatilaka et al., 2007]

- Gaussian noise model [K.D. Jarman et al., 2011]
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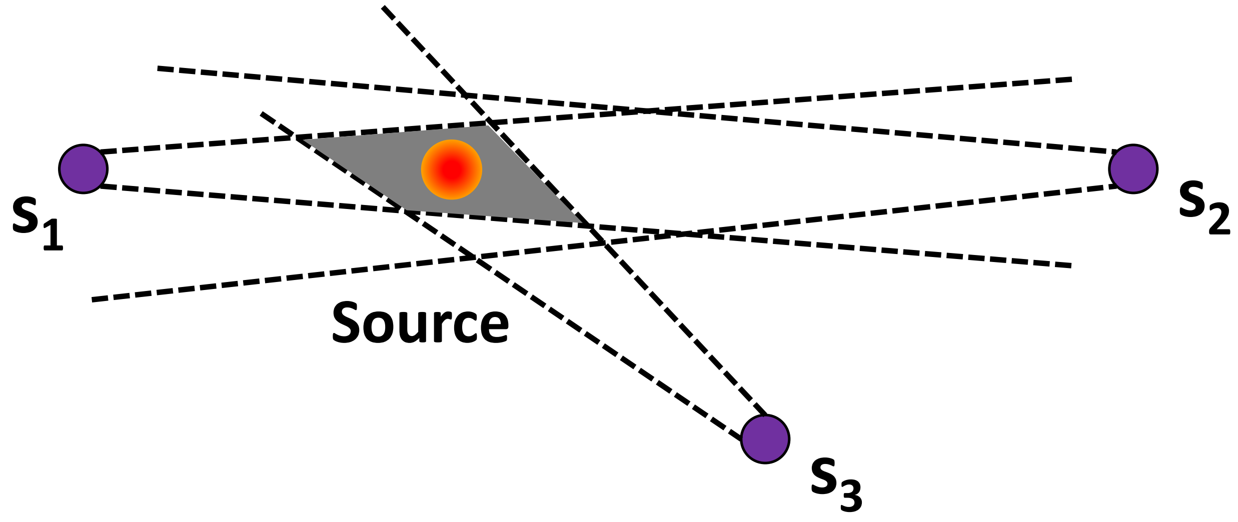
Three sensors are sufficient for localizing the source



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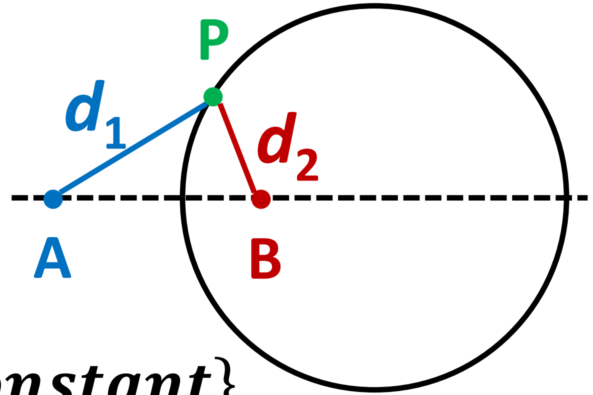
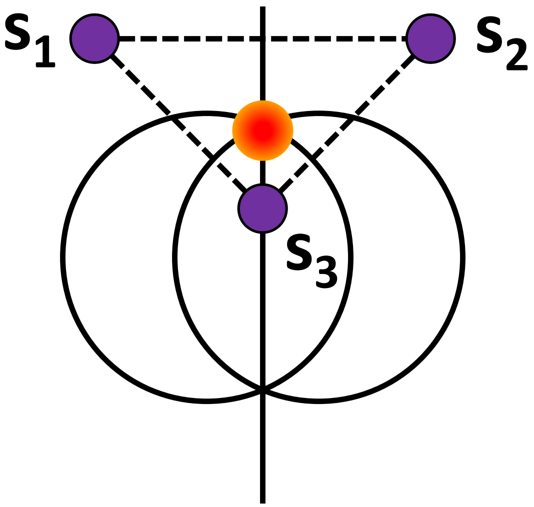
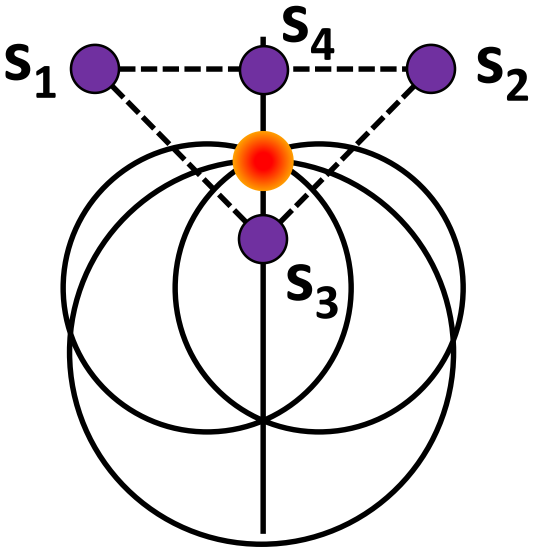
Four sensors are sufficient for localizing the source

Apollonius' definition of a circle:



$$\{P|\, d_1/d_2 = constant\}$$





Related Work

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- Angle-base (Mean of Estimates) [D. Niculescu et al., 2003]
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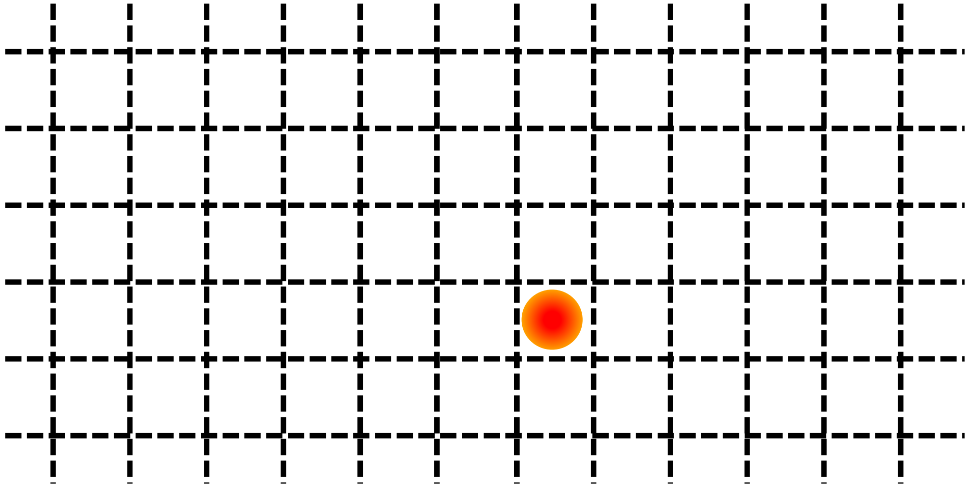
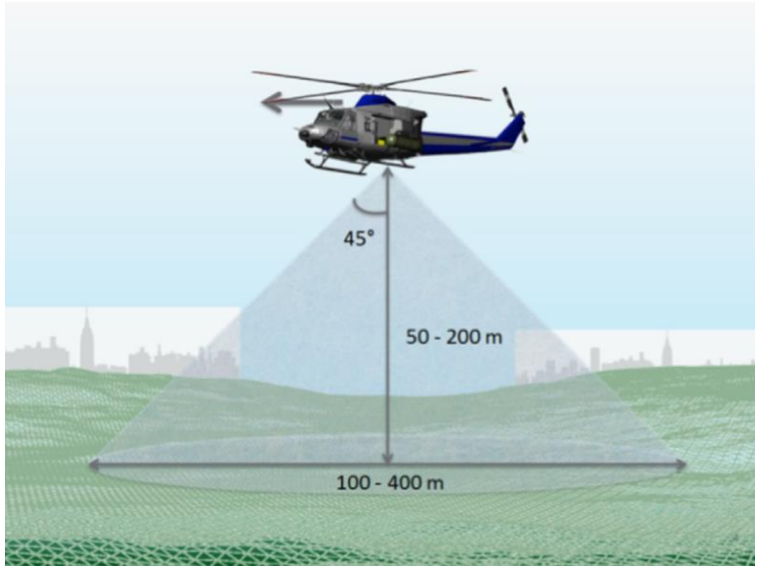
- **Maximum count rate (stationary source) [D.K. Fagan et al., 2012]**

Exhaustive search in a area





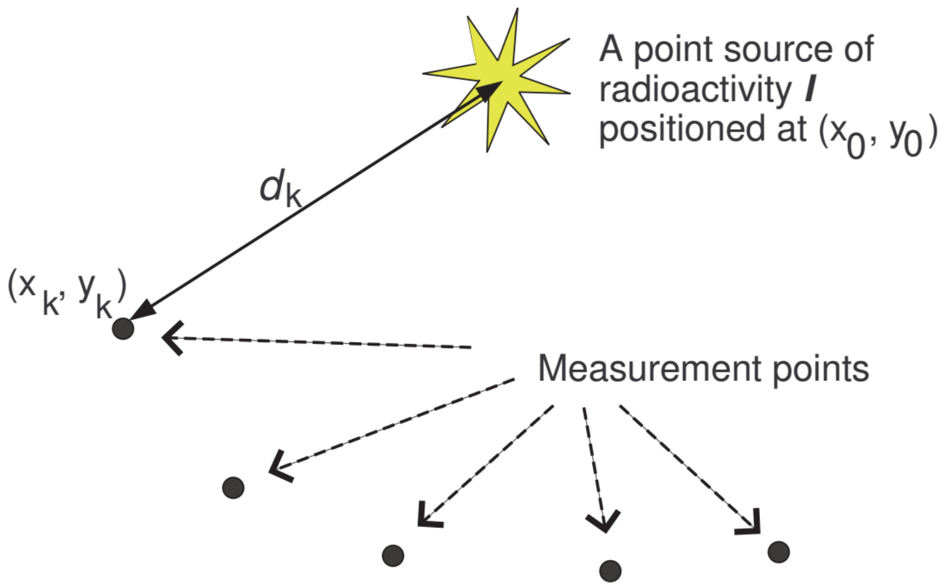
Aerial detection

Related Work

Model-based:

Maximum Likelihood Estimation (MLE) [A. Gunatilaka et al., 2007]

- **Gaussian noise model [K.D. Jarman et al., 2011]**
- **Poisson noise model [M. Wieneke et al., 2012]**



1) **Assume an parametric model of count rate and distance:**

$$\mu_k(x_0, y_0) = \frac{I}{(x_k - x_0)^2 + (y_k - y_0)^2} + b$$

2) **Assume Gaussian noise:**

$$T_k \sim \mathcal{N}(\mu_k, \mu_k)$$

3) **Maximize the likelihood:**

$$[\hat{x}_0, \hat{y}_0] = \arg\max_{x_0, y_0} P(T_1, T_2, \cdots, T_k | \mu)$$

Related Work

Model-based:

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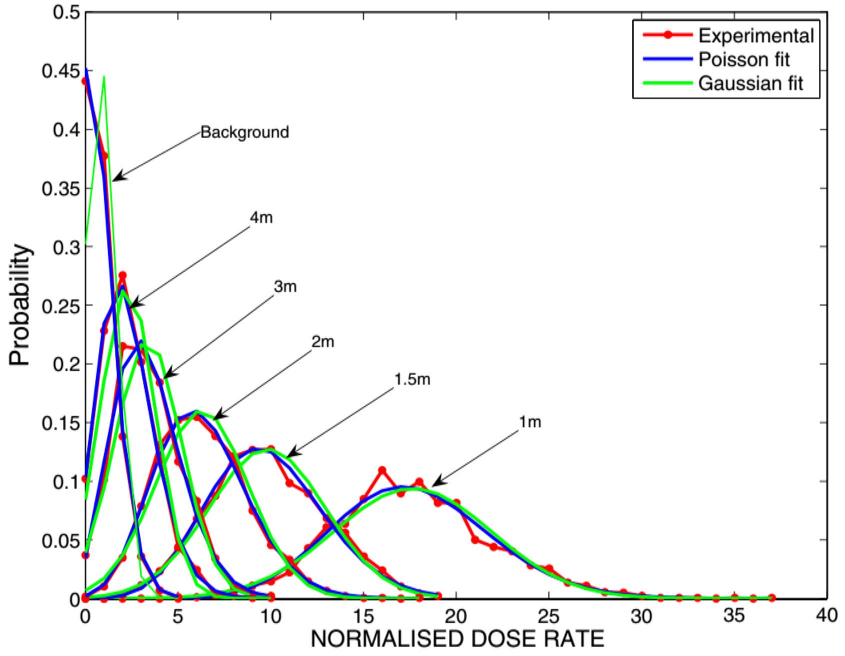


The only difference is in the 2nd step, assuming Poisson noise:

$$P(T_k; \lambda = \mu_k) = \frac{e^{-\mu_k} \cdot \mu_k^{T_k}}{T_k!}$$

Related Work

Related work:

- Scattered detectors
- Parametric model
- Gaussian noise
- Maximum likelihood

Ours approach:

- Structured detectors
- Non-parametric model
- Gaussian noise
- Maximum likelihood (1NN)

Our Approach and Results

The data we have:

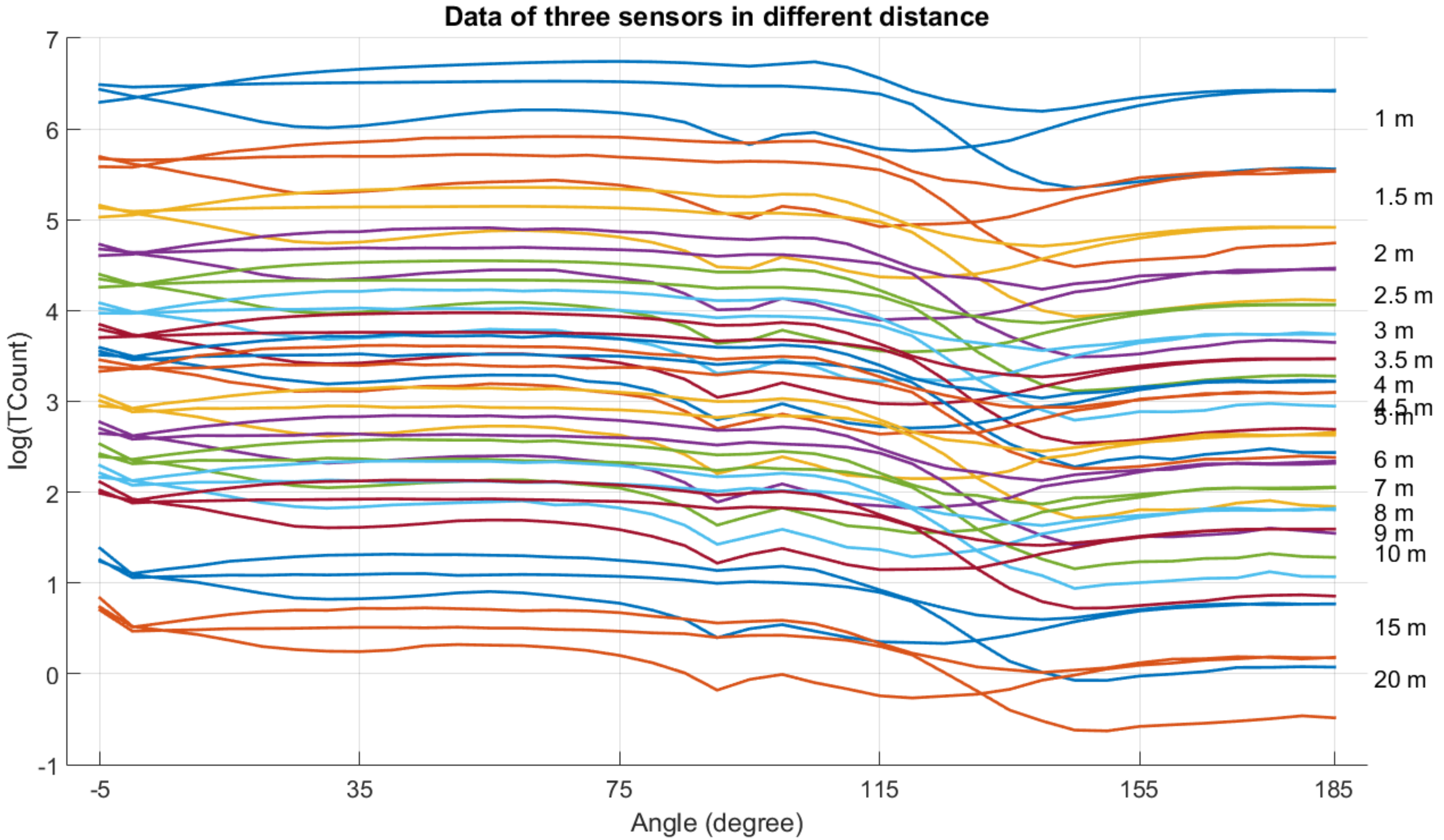
- **Angles:**

  -5 ~ 185 degree with increment of 5 degree.

- **Distances:**
  - **1 ~ 5m with increment of 0.5m;**
  - **6~10m with increment of 1m;**
  - **15 and 20m.**

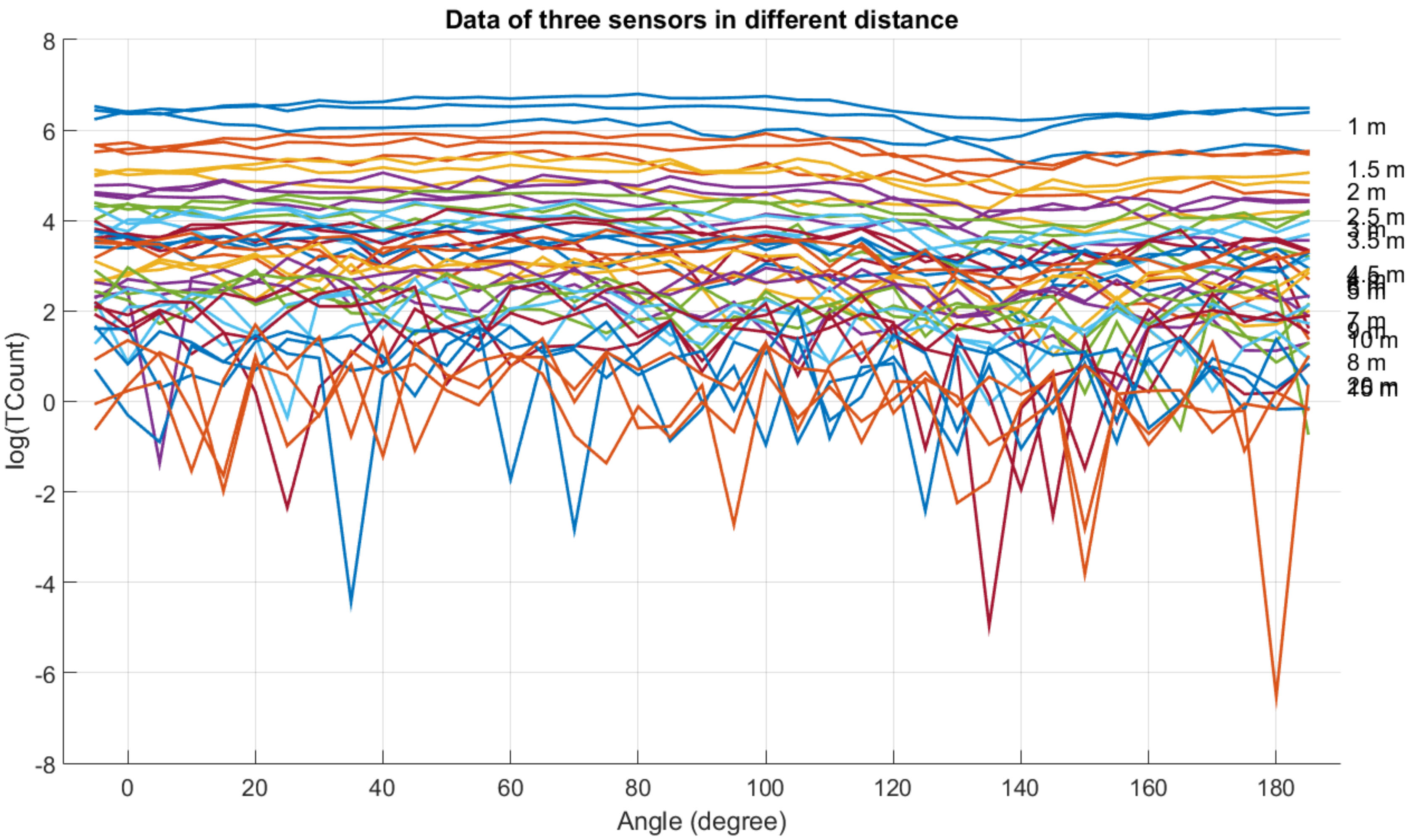
Our Approach and Results

The raw data

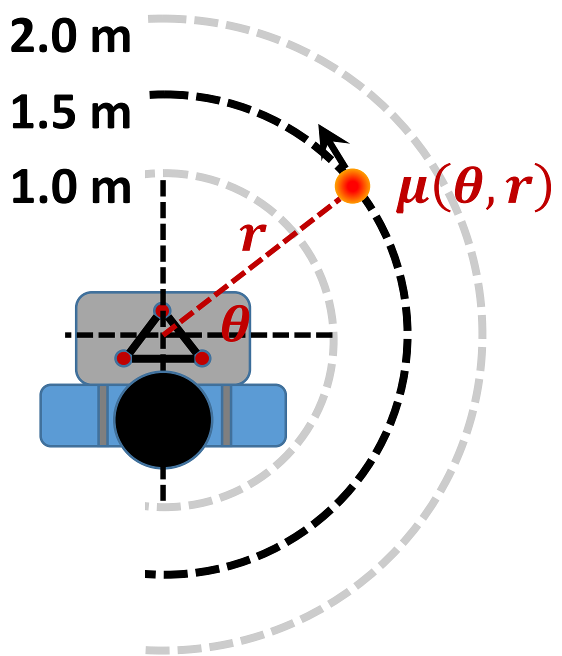


Our Approach and Results

The noisy data



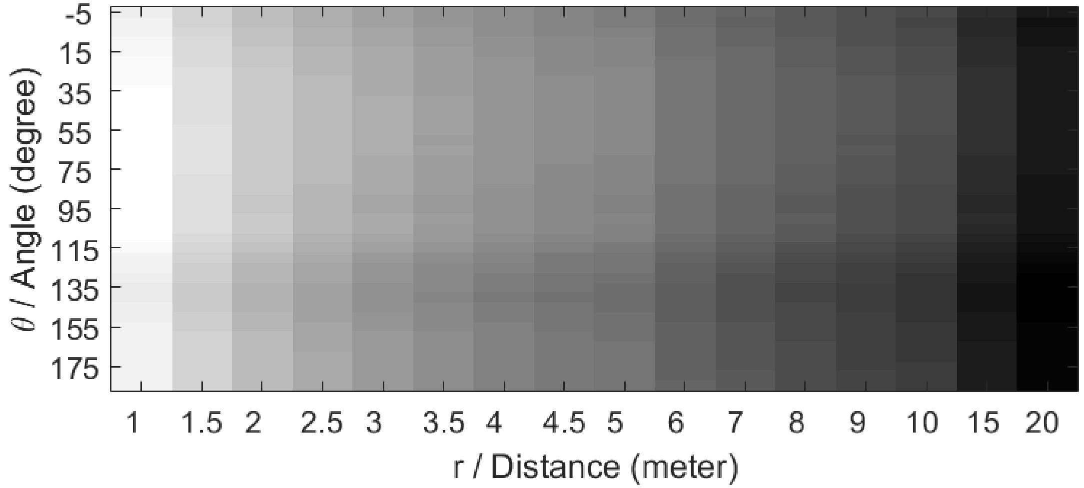
Our Approach and Results



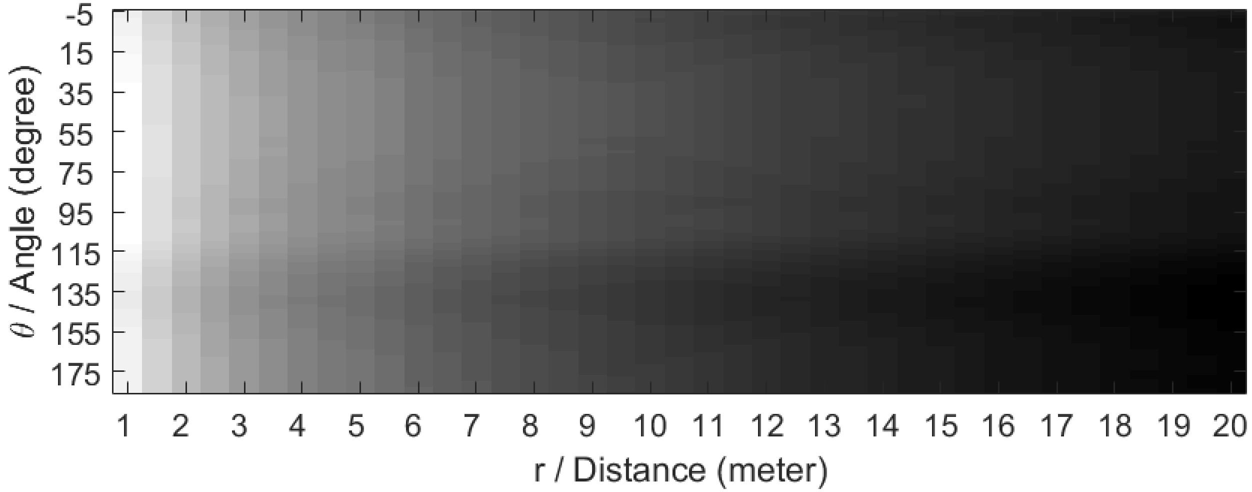
Step 1: Construct $\mu_i(\theta, r)$, $i = 1, 2, 3$ (assume three detectors):

1) Interpolation (regression) on both θ and r
2) Build 2-D lookup table (angle vs. distance)

Raw data of the *i*th detector



$\mu_i(\theta, r)$ after interpolation



Our Approach and Results

Step 2: Assume Gaussian noise: $T_i \sim \mathcal{N}\big(\mu_i(\theta, r), \mu_i(\theta, r)\big)$

$$P\big(T_i | \mu_i(\theta, r)\big) = \frac{1}{\sqrt{2\pi\mu_i(\theta, r)}} e^{-\frac{\left(T_i - \mu_i(\theta, r)\right)^2}{2\mu_i(\theta, r)}}$$

Step 3: Maximum likelihood estimation:

$$\left[\hat{\theta}, \hat{r}\right] = \arg\max_{\theta, r} P(T_1, T_2, T_3 | \mu_1, \mu_2, \mu_3)$$

Our Approach and Results

Assume the three detectors are independent,

$$P(T_1, T_2, T_3 | \mu_1, \mu_2, \mu_3)$$

$$= P(T_1|\mu_1, T_2|\mu_2, T_3|\mu_3)$$

$$= P(T_1|\mu_1)P(T_2|\mu_2)P(T_3|\mu_3)$$

$$= \sum_{i=1}^{3} \frac{1}{\sqrt{2\pi\mu_i}} \exp\left(-\frac{(T_i - \mu_i)^2}{2\mu_i}\right)$$

Log-likelihood:

$$= -\frac{1}{2}\sum_{i=1}^{3} \log(2\pi\mu_i) - \frac{1}{2}\sum_{i=1}^{3} \frac{(T_i - \mu_i)^2}{\mu_i}$$

Finally,

$$\arg\min_{\theta, r} \left(\sum_{i=1}^{3} \log \mu_i + \sum_{i=1}^{3} \frac{(T_i - \mu_i)^2}{\mu_i} \right)$$

Our Approach and Results

In practice, we may have only one sample for each $(\boldsymbol{\theta}, \boldsymbol{r})$ pair.

$$\arg\min_{\theta, r} \left(\sum_{i=1}^{3} \log \boxed{\mu_i} + \sum_{i=1}^{3} \frac{(T_i - \mu_i)^2}{\boxed{\mu_i}} \right)$$

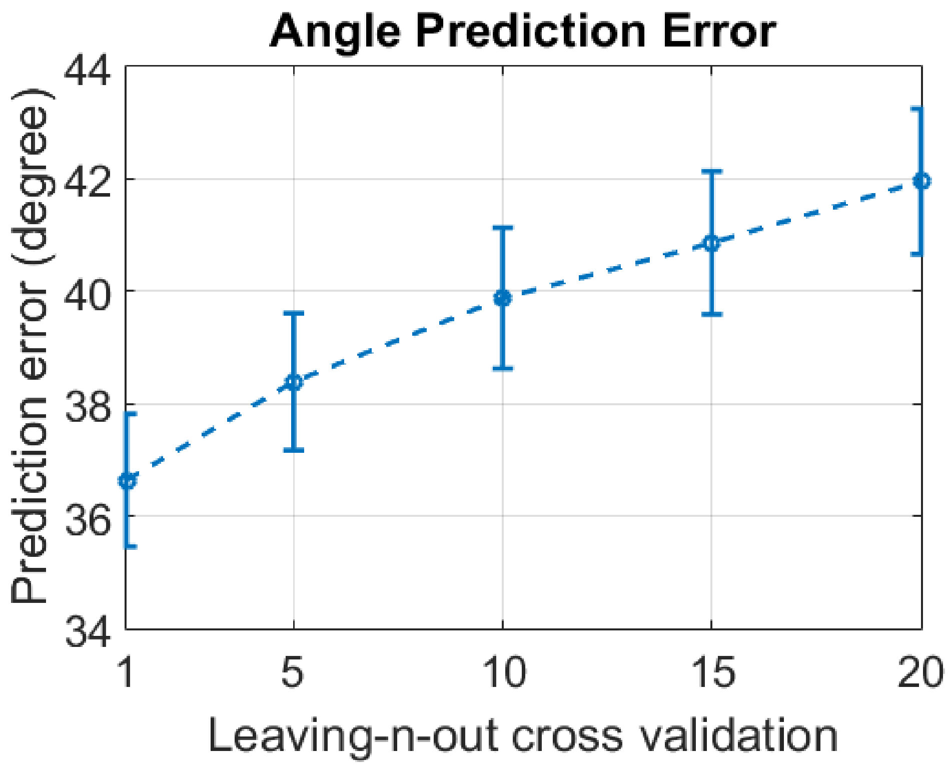
$$\sigma^2$$

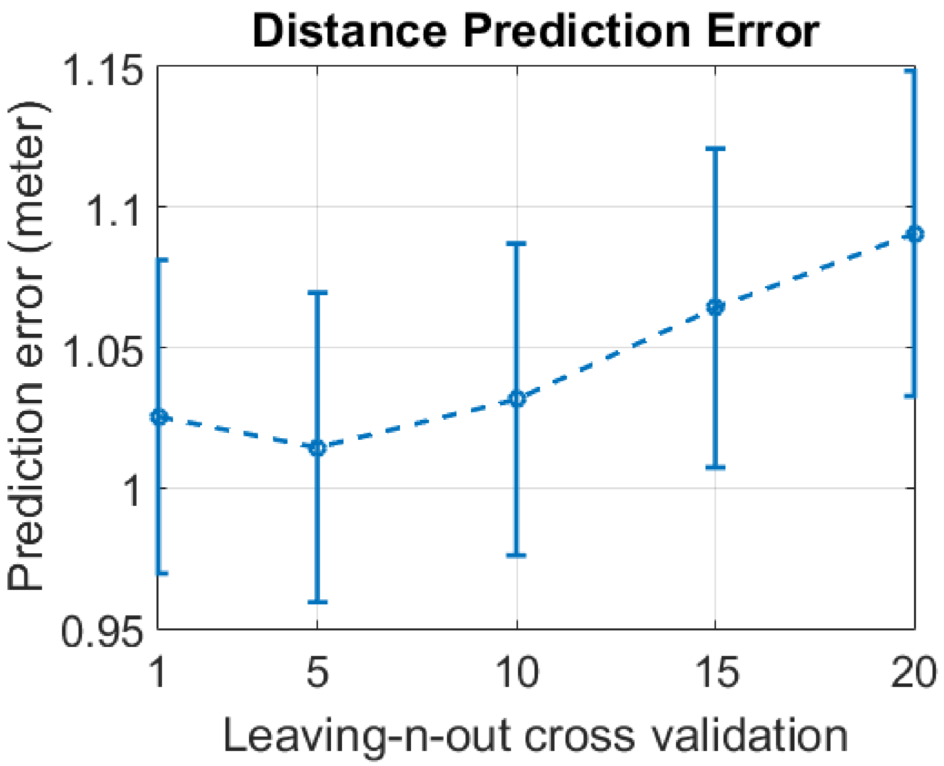
Equivalent to 1NN:

$$\arg\min_{\theta, r} \sum_{i=1}^{3} (T_i - \mu_i)^2$$

Our Approach and Results

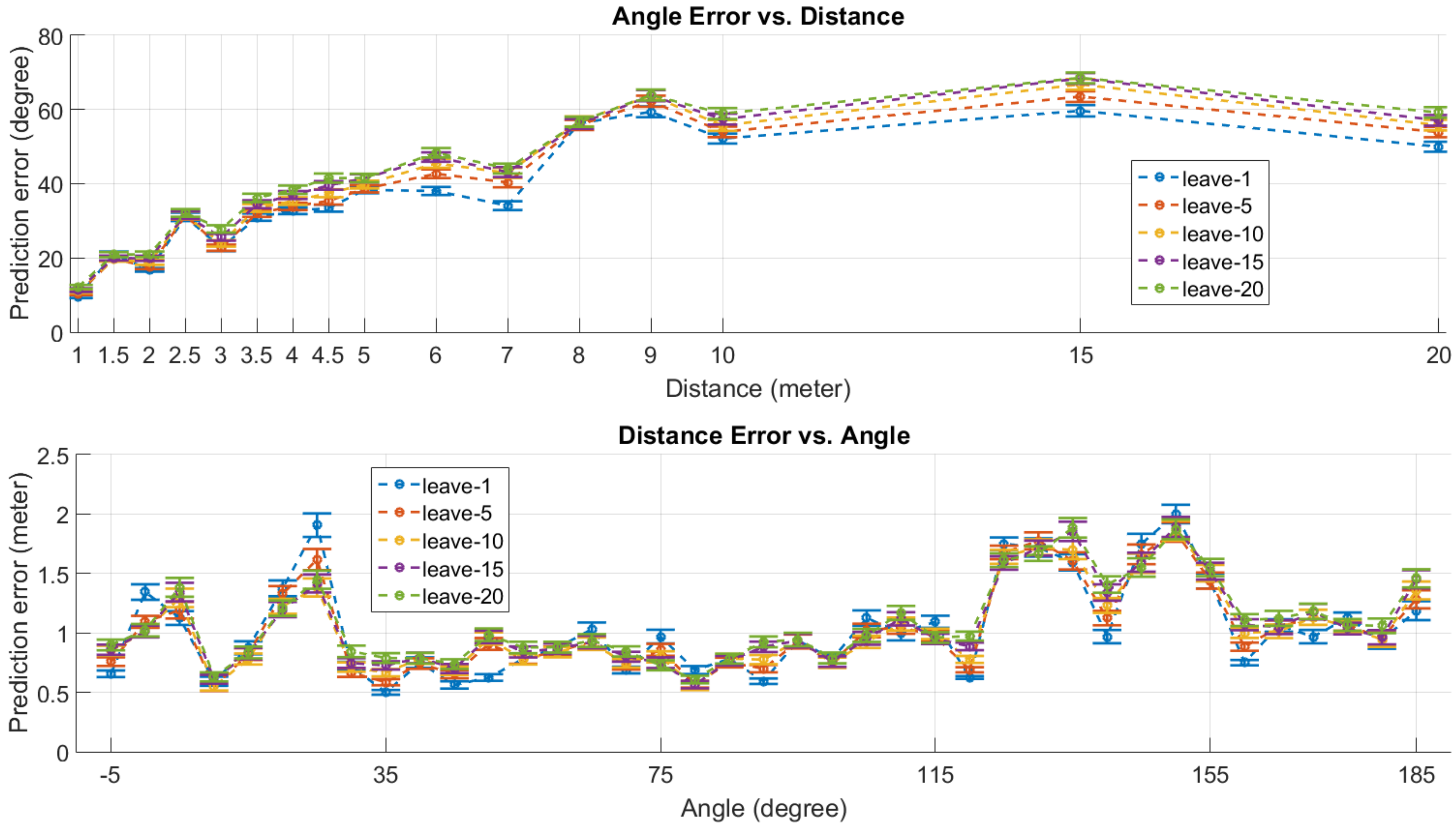
Random leave-n-out cross validation, 1000 iteration:





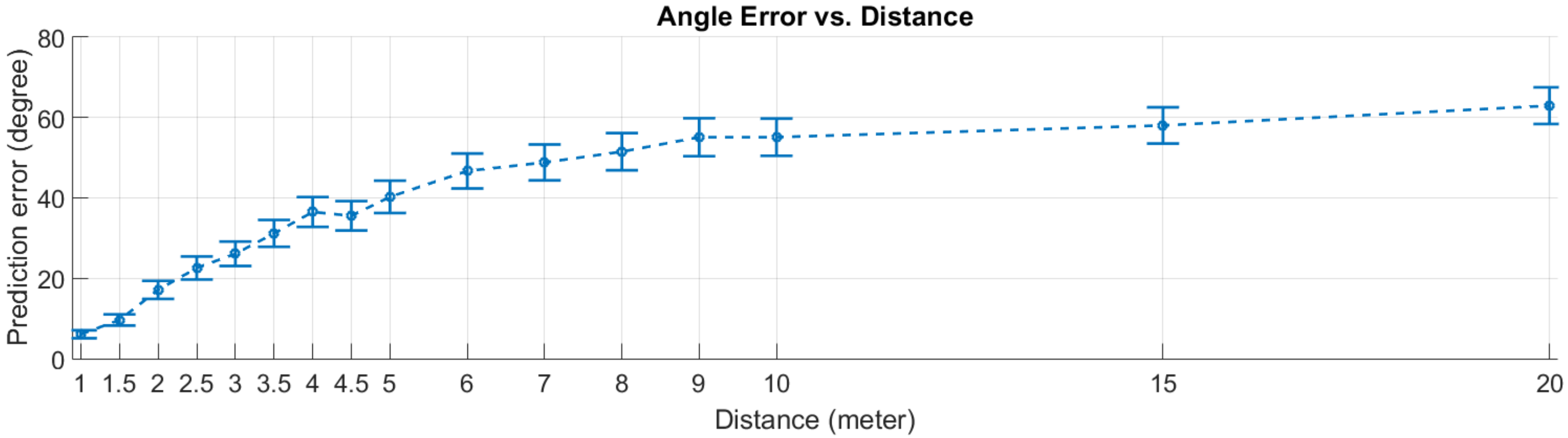
Our Approach and Results

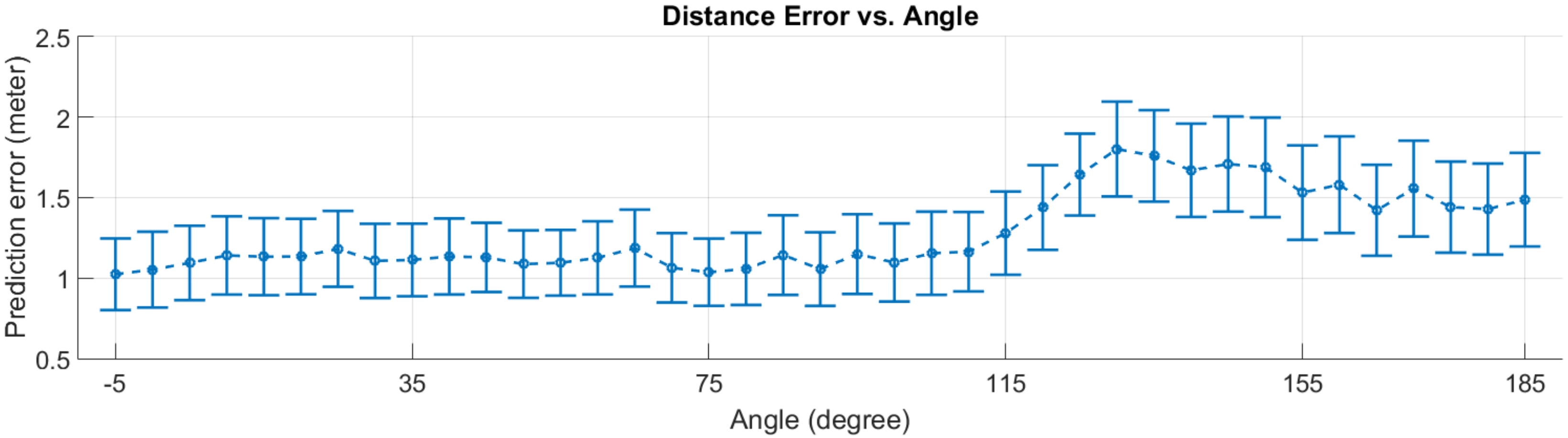
Random leave-n-out cross validation, apply 1NN 1000 iteration :



Our Approach and Results

If there are enough samples to estimate $\mu_i(\theta, r)$, apply MLE:





Thank You